Fordelingsfunktion for normalfordelingen

30. marts 2020

$Z = \sigma X + \mu$ har fordelingsfunktionen

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv$$

$X \sim N(0,1)$ har fordelingsfunktionen

$$\Phi(x) = P(X \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$$

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Først udnyttes at $Z = \sigma X + \mu$.

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$$F(x) = P(Z \leq x)$$

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$$\begin{aligned} F(x) &= P(Z \leq x) \\ &= P(\sigma X + \mu \leq x) \end{aligned}$$

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$$\begin{aligned} F(x) &= P(Z \le x) \\ &= P(\sigma X + \mu \le x) \\ &= P(\sigma X \le x - \mu) \\ &= P\left(X \le \frac{x-\mu}{\sigma}\right) \end{aligned}$$

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$$\begin{aligned} F(x) &= P(Z \leq x) \\ &= P(\sigma X + \mu \leq x) \\ &= P(\sigma X \leq x - \mu) \\ &= P\left(X \leq \frac{x-\mu}{\sigma}\right) \\ &= \Phi\left(\frac{x-\mu}{\sigma}\right) \end{aligned}$$

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$$\Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}t^2} dt$$

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$v = \sigma t + \mu$ og derfor er $\dfrac{dv}{dt} = \sigma$ og ved isolering af dt fås, at

$$dt = \frac{1}{\sigma} dv$$

$$\Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}t^2} dt$$
$$= \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} \frac{1}{\sigma} dv$$

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$v = \sigma t + \mu$

Da $v = \sigma t + \mu$ kan t isoleres.

$$v = \sigma t + \mu \Leftrightarrow v - \mu = \sigma t \Leftrightarrow \frac{v-\mu}{\sigma} = t$$

$$\Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}t^2} dt$$

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$$\begin{aligned}
\Phi\left(\frac{x-\mu}{\sigma}\right) &= \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}t^2} dt \\
&= \frac{1}{\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} \frac{1}{\sigma} dv \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv
\end{aligned}$$

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$v = \sigma t + \mu$

Den nedre grænse $t = -\infty$ og da σ positiv vil også $v = \sigma t + \mu = -\infty$. Den øvre grænse er $t = \dfrac{x-\mu}{\sigma}$. Dette indsættes i $v = \sigma t + \mu$, for at beregne den tilsvarende v-værdi.

$$v = \sigma\frac{x-\mu}{\sigma} + \mu = x - \mu + \mu = x$$

$$\begin{aligned}
\Phi\left(\frac{x-\mu}{\sigma}\right) &= \frac{1}{\sqrt{2\pi}}\int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}t^2} dt \\
&= \frac{1}{\sqrt{2\pi}}\int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} \frac{1}{\sigma} dv \\
&= \frac{1}{\sigma\sqrt{2\pi}}\int_{t=-\infty}^{t=\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv \\
&= \frac{1}{\sigma\sqrt{2\pi}}\int_{v=-\infty}^{v=x} e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv
\end{aligned}$$