Symmetri af differentiationsrækkefølge

Bevis for at $f''_{xy}(x, y) = f''_{yx}(x, y)$
i $P(a, b, f(a, b))$

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$$\begin{aligned} Q &= f(a+h,b+k) - f(a+h,b) \\ &\quad - f(a,b+k) + f(a,b) \end{aligned}$$

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&\quad -f(a, b+k) + f(a, b) \\
u(x) &= f(x, b+k) - f(x, b) \\
v(y) &= f(a+h, y) - f(a, y)
\end{aligned}$$

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$$Q = u(a+h) - u(a)$$

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$$Q = u(a+h) - u(a)$$

$$Q = v(b+k) - v(b)$$

$$\frac{u(a+h)-u(a)}{(a+h)-a} = u'(a+i_1\cdot h), \quad 0 \le i_1 < 1$$

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$$\frac{u(a+h)-u(a)}{h} = u'(a+i_1 \cdot h)$$

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$$Q = u(a+h) - u(a)$$

$$Q = v(b+k) - v(b)$$

$$\begin{aligned} \frac{u(a+h)-u(a)}{(a+h)-a} &= u'(a+i_1\cdot h), \quad 0 \le i_1 < 1 \\ \frac{u(a+h)-u(a)}{h} &= u'(a+i_1\cdot h) \\ u(a+h)-u(a) &= h\cdot u'(a+i_1\cdot h) \end{aligned}$$

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a + i_1 \cdot h) \end{aligned}$$

$$Q = v(b+k) - v(b)$$

$$\frac{u(a+h)-u(a)}{(a+h)-a} = u'(a+i_1\cdot h), \quad 0 \le i_1 < 1$$

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a + i_1 \cdot h) \\ &= h \cdot \left(f'_x(a + i_1 \cdot h, b+k) - f'_x(a + i_1 \cdot h, b) \right) \end{aligned}$$

$$Q = v(b+k) - v(b)$$

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a+i_1 \cdot h) \\ &= h \cdot \left(f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b) \right) \end{aligned}$$

$$Q = v(b+k) - v(b)$$

$$\frac{f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b)}{(b+k) - b}$$
$$= f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k)$$

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a + i_1 \cdot h) \\ &= h \cdot \left(f'_x(a + i_1 \cdot h, b+k) - f'_x(a + i_1 \cdot h, b)\right) \end{aligned}$$

$$Q = v(b+k) - v(b)$$

$$\frac{f'_x(a+i_1\cdot h,b+k) - f'_x(a+i_1\cdot h,b)}{(b+k)-b} = f''_{xy}(a+i_1\cdot h, b+i_2\cdot k)$$

$$\frac{f'_x(a+i_1\cdot h,b+k) - f'_x(a+i_1\cdot h,b)}{k} = f''_{xy}(a+i_1\cdot h, b+i_2\cdot k)$$

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$$\begin{aligned} Q &= f(a+h,b+k) - f(a+h,b) \\ &\quad -f(a,b+k) + f(a,b) \\ u(x) &= f(x,b+k) - f(x,b) \\ v(y) &= f(a+h,y) - f(a,y) \end{aligned}$$

$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a+i_1 \cdot h) \\ &= h \cdot \left(f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b)\right) \end{aligned}$$

$$Q = v(b+k) - v(b)$$

$$\frac{f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b)}{(b+k)-b} = f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k)$$

$$\frac{f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b)}{k} = f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k)$$

$$f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b) = k \cdot f''_{xy}(a+i_1 \cdot h, b+i \cdot k)$$

Symmetri af differentiationsrækkefølge

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a+i_1 \cdot h) \\ &= h \cdot \left(f'_x(a+i_1\cdot h, b+k) - f'_x(a+i_1\cdot h, b)\right) \\ &= h \cdot \left(k \cdot f''_{xy}(a+i_1\cdot h, b+i_2\cdot k)\right) \\ Q &= v(b+k) - v(b) \end{aligned}$$

$$\frac{f'_x(a+i_1\cdot h, b+k) - f'_x(a+i_1\cdot h, b)}{(b+k)-b} = f''_{xy}(a+i_1\cdot h, b+i_2\cdot k)$$

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$$\begin{aligned} Q &= f(a+h,b+k) - f(a+h,b) \\ &\quad - f(a,b+k) + f(a,b) \\ u(x) &= f(x,b+k) - f(x,b) \\ v(y) &= f(a+h,y) - f(a,y) \end{aligned}$$

$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a + i_1 \cdot h) \\ &= h \cdot \left(f'_x(a + i_1 \cdot h, b+k) - f'_x(a + i_1 \cdot h, b) \right) \\ &= h \cdot \left(k \cdot f''_{xy}(a + i_1 \cdot h, b + i_2 \cdot k) \right) \\ Q &= v(b+k) - v(b) \\ &= k \cdot \left(h \cdot f''_{yx}(a + i_4 \cdot h, b + i_3 \cdot k) \right) \end{aligned}$$

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a+i_1 \cdot h) \\ &= h \cdot \left(f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b)\right) \\ &= h \cdot \left(k \cdot f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k)\right) \\ Q &= v(b+k) - v(b) \\ &= k \cdot \left(h \cdot f''_{yx}(a+i_4 \cdot h, b+i_3 \cdot k)\right) \end{aligned}$$

$$f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k) = f''_{yx}(a+i_3 \cdot h, b+i_4 \cdot k)$$

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$$\begin{aligned} Q &= u(a+h) - u(a) \\ &= h \cdot u'(a+i_1 \cdot h) \\ &= h \cdot \left(f'_x(a+i_1 \cdot h, b+k) - f'_x(a+i_1 \cdot h, b)\right) \\ &= h \cdot \left(k \cdot f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k)\right) \\ Q &= v(b+k) - v(b) \\ &= k \cdot \left(h \cdot f''_{yx}(a+i_4 \cdot h, b+i_3 \cdot k)\right) \end{aligned}$$

$$\begin{aligned} f''_{xy}(a+i_1 \cdot h, b+i_2 \cdot k) &= f''_{yx}(a+i_3 \cdot h, b+i_4 \cdot k) \\ f''_{xy}(a,b) &= f''_{yx}(a,b) \end{aligned}$$