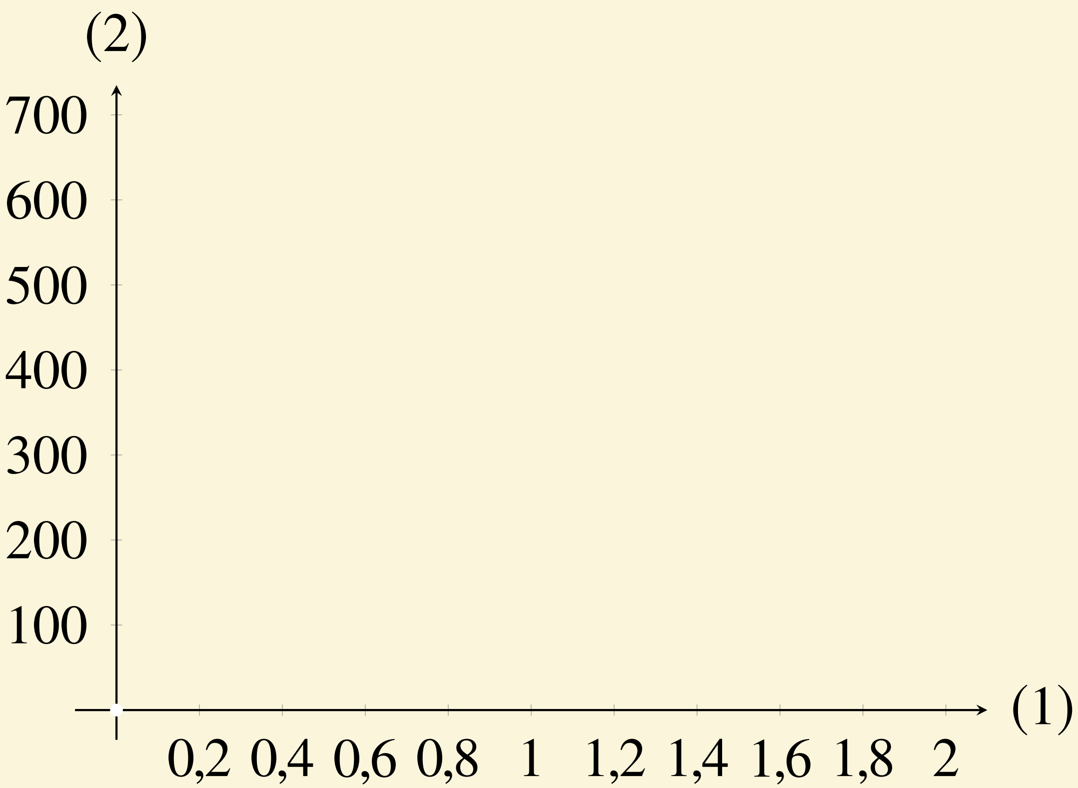
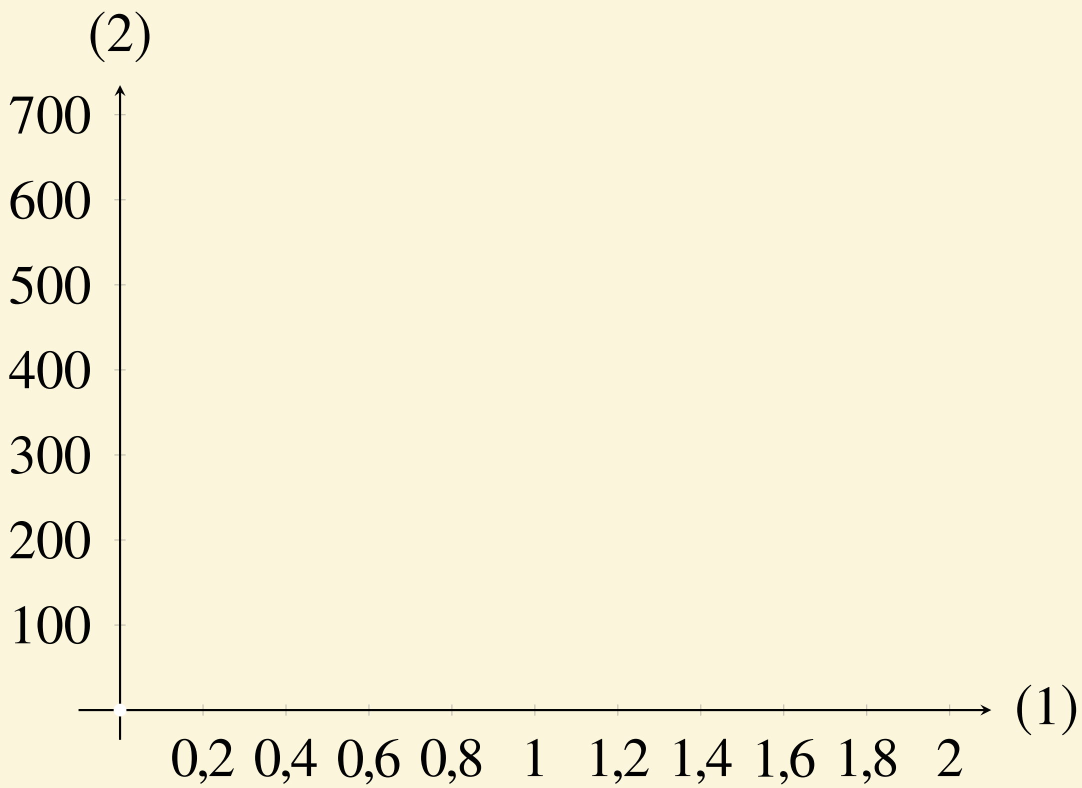
Numerisk løsning af differentialligning

$$\frac{dy}{dt} = y$$



$$\frac{dy}{dt} = y$$

$$t_n = n \cdot \Delta t$$



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Numerisk løsning af differentialligning

17. marts 2020

$$\frac{dy}{dt} = y$$

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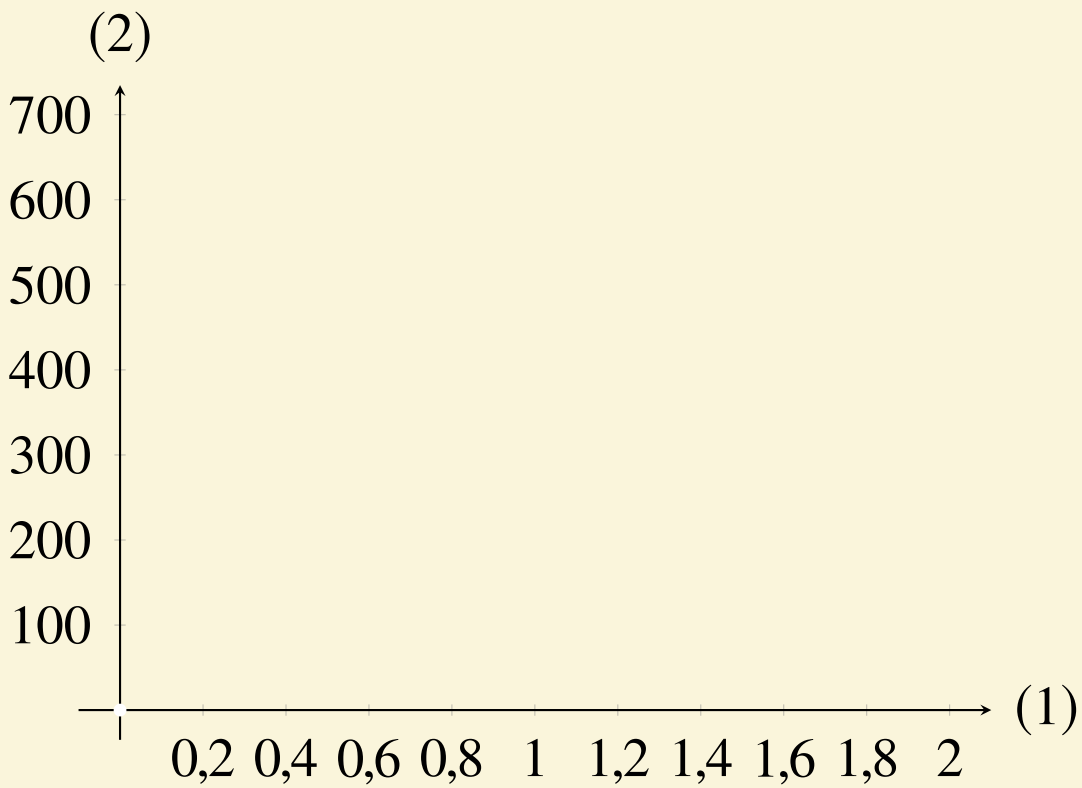

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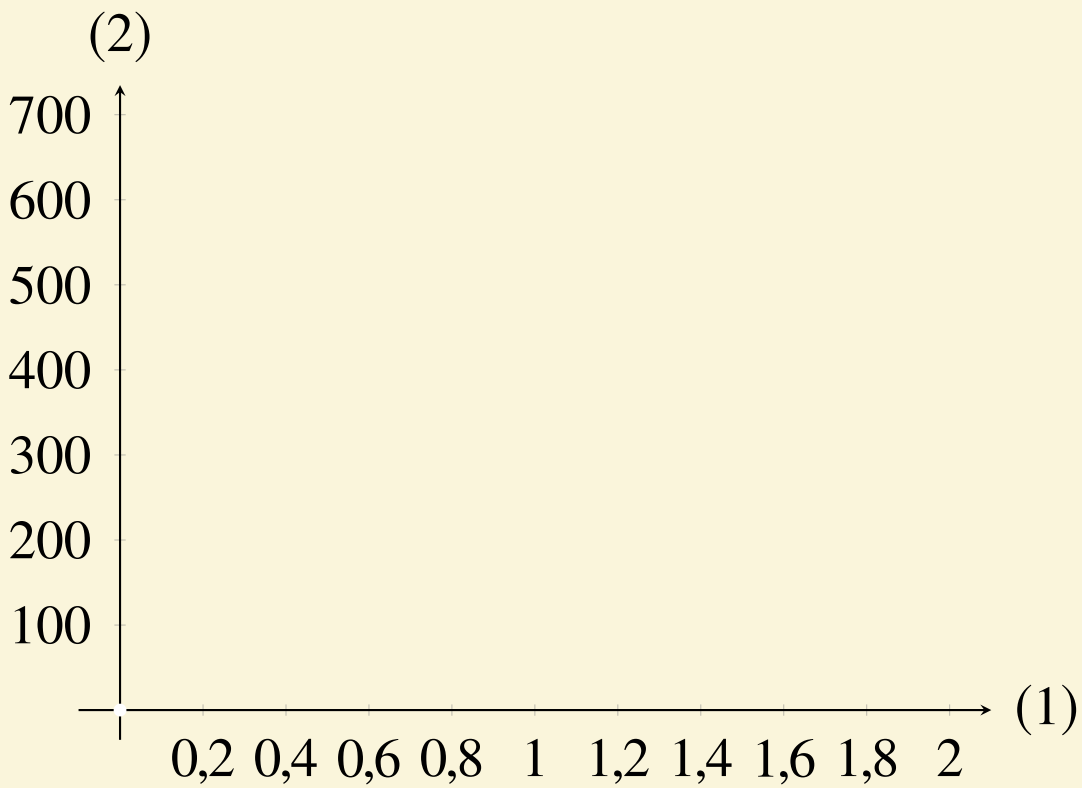
Lad $y_0 = 100$ og lad $\Delta t = 0.5$.



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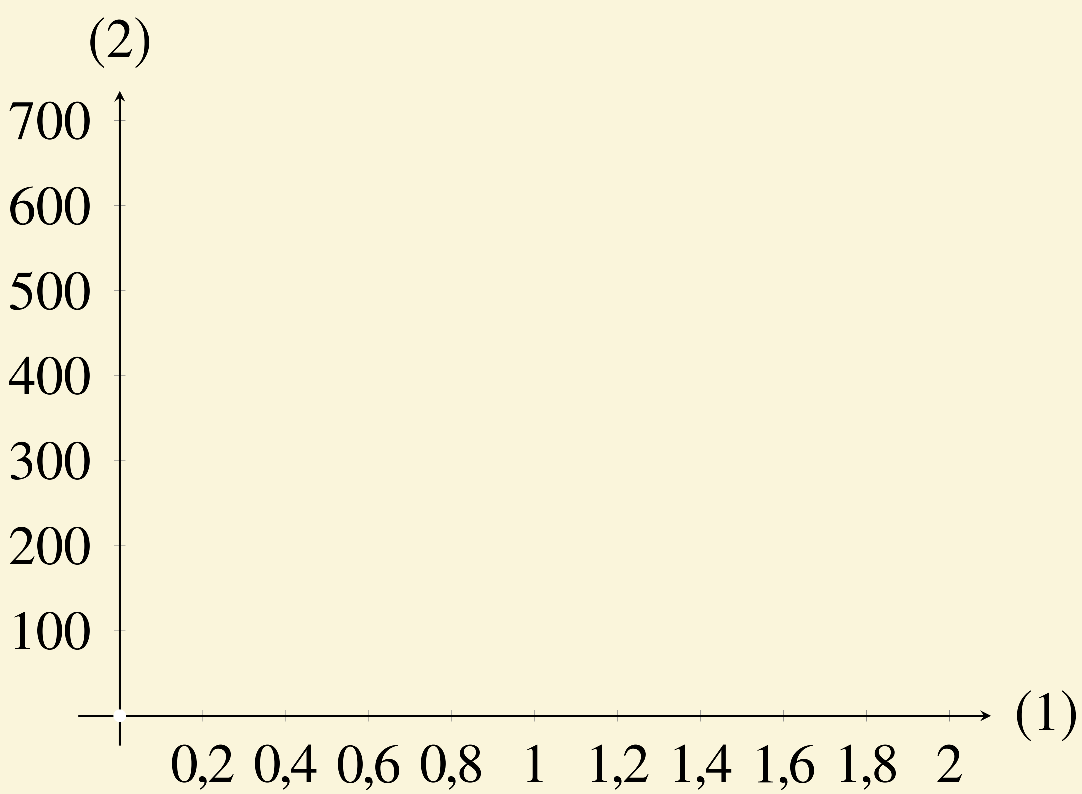

$$y_0 = 100$$

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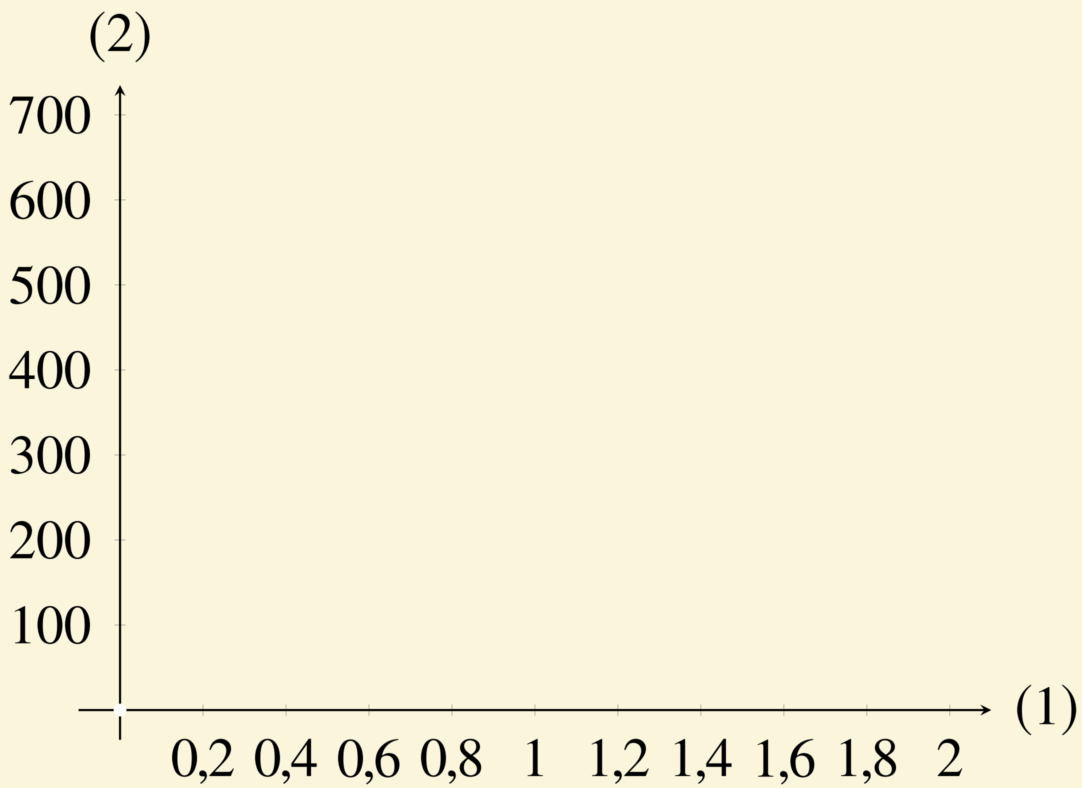

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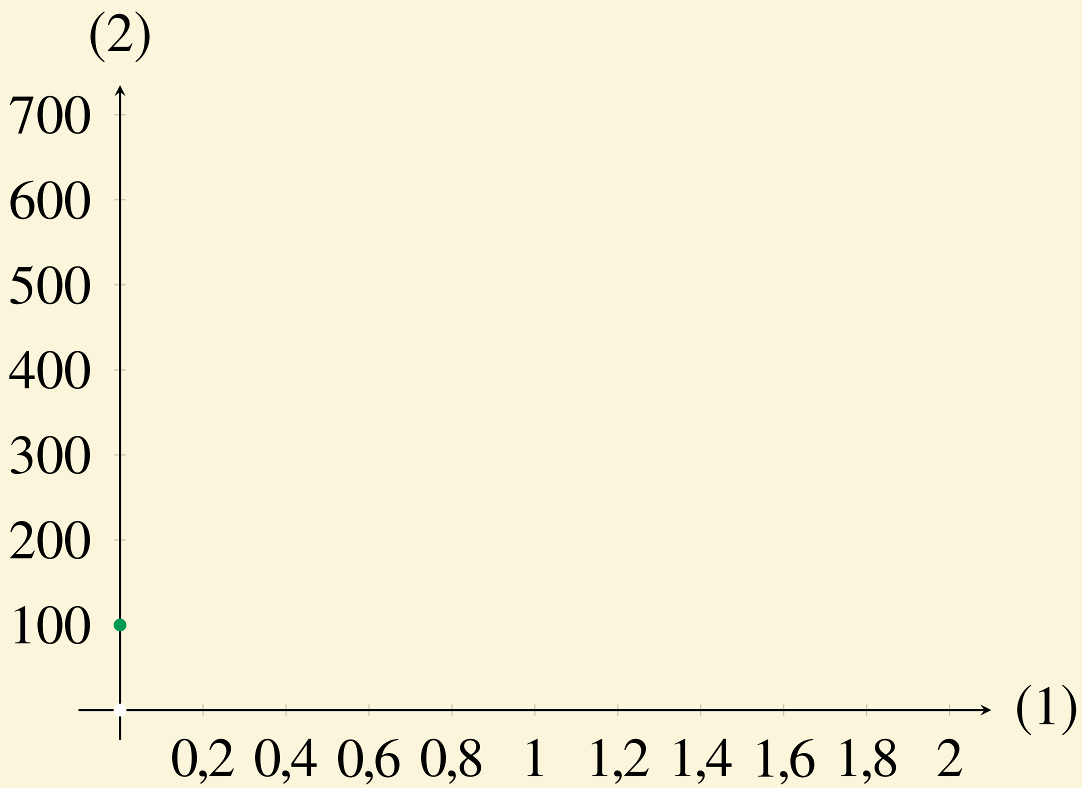

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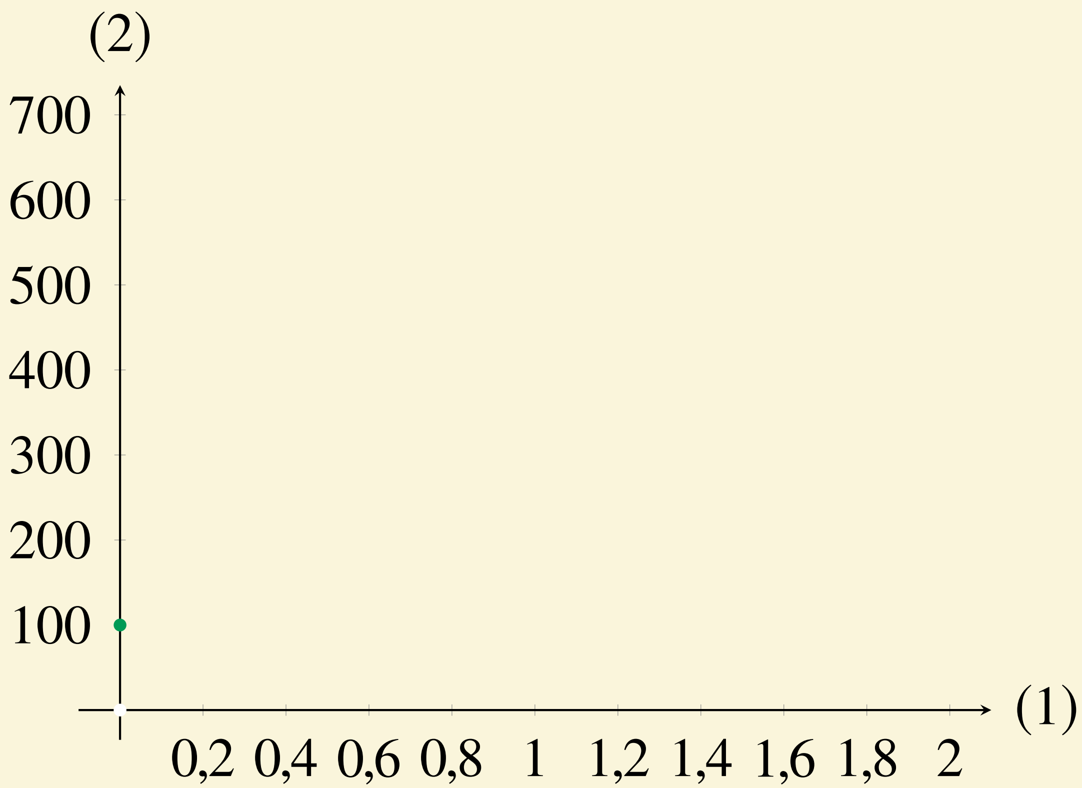

$$\begin{aligned} y_0 &= 100 \\ t_0 &= 0 \cdot 0.5 = 0 \\ y_1 &= 100 + 100 \cdot 0.5 = 150 \end{aligned}$$

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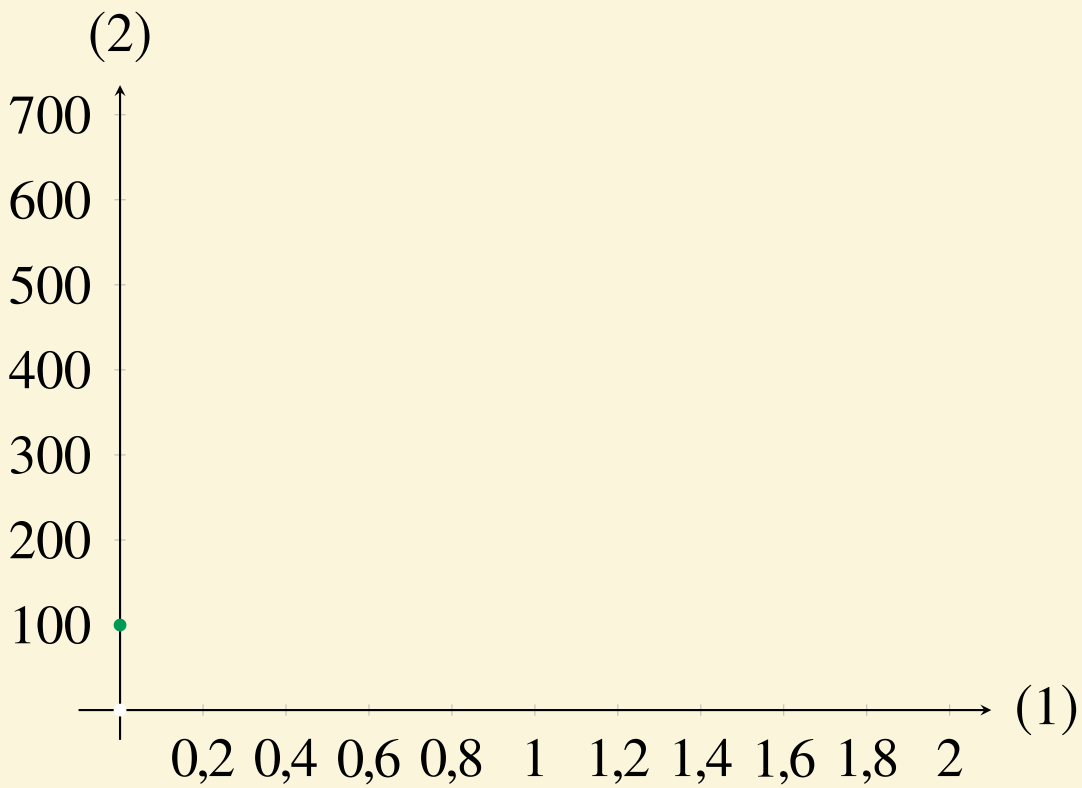

$$\begin{aligned} y_0 &= 100 \\ t_0 &= 0 \cdot 0.5 = 0 \\ y_1 &= 100 + 100 \cdot 0.5 = 150 \\ t_1 &= 1 \cdot 0.5 = 0.5 \end{aligned}$$

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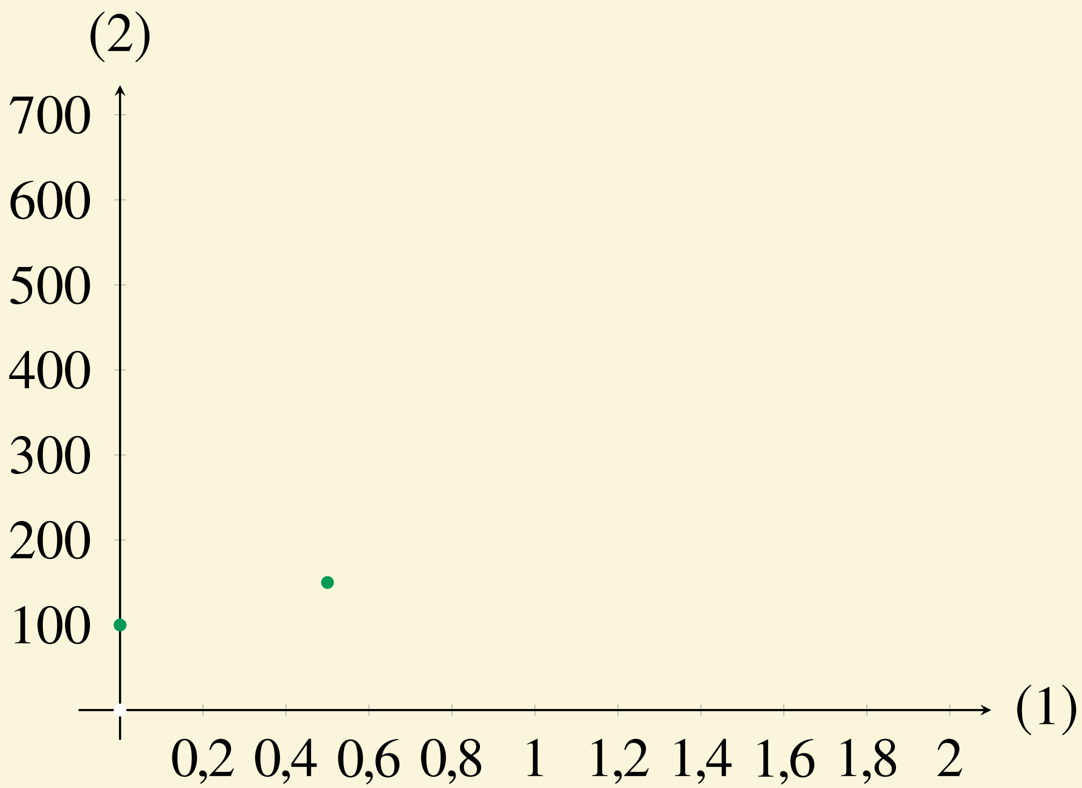

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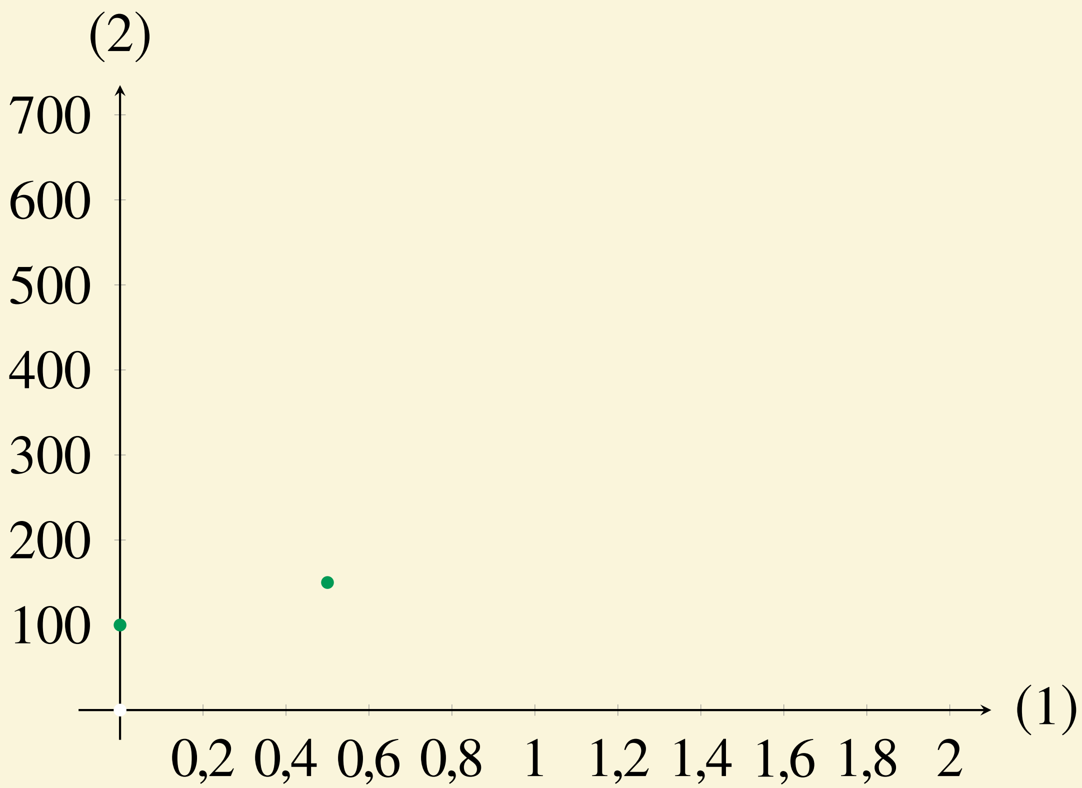

$$\begin{aligned} y_0 &= 100 \\ t_0 &= 0 \cdot 0.5 = 0 \\ y_1 &= 100 + 100 \cdot 0.5 = 150 \\ t_1 &= 1 \cdot 0.5 = 0.5 \\ y_2 &= 150 + 150 \cdot 0.5 = 225 \end{aligned}$$

Numerisk løsning af differentialligning

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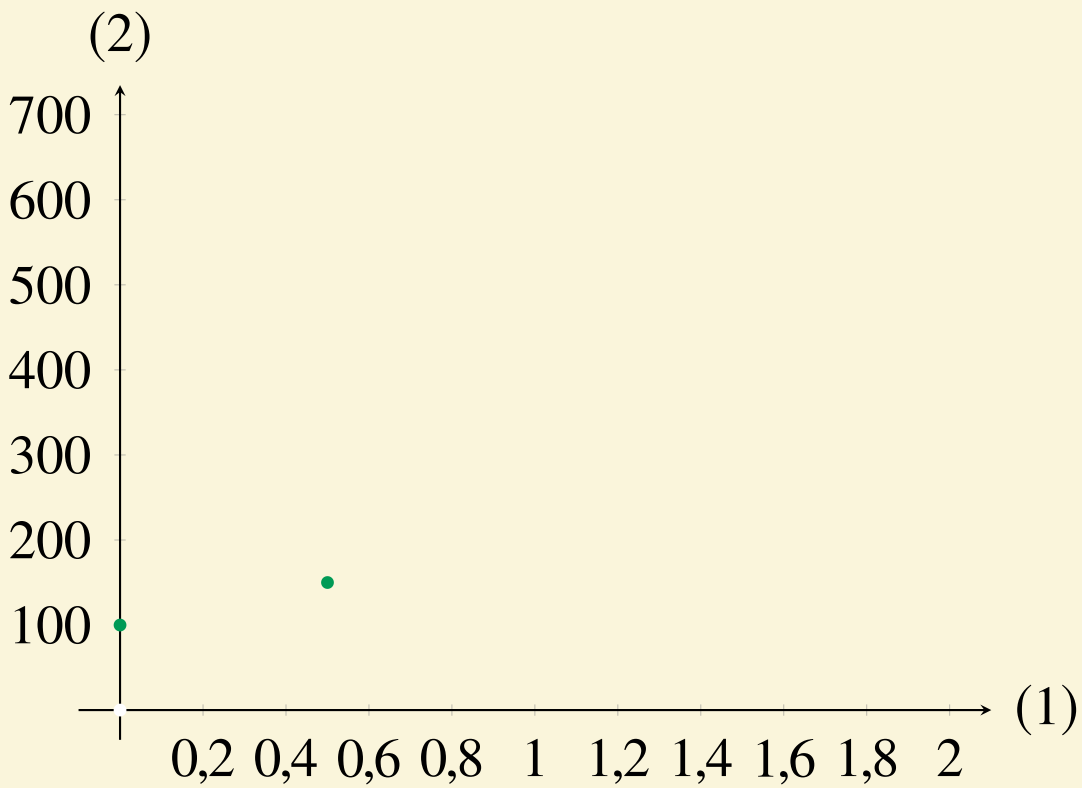

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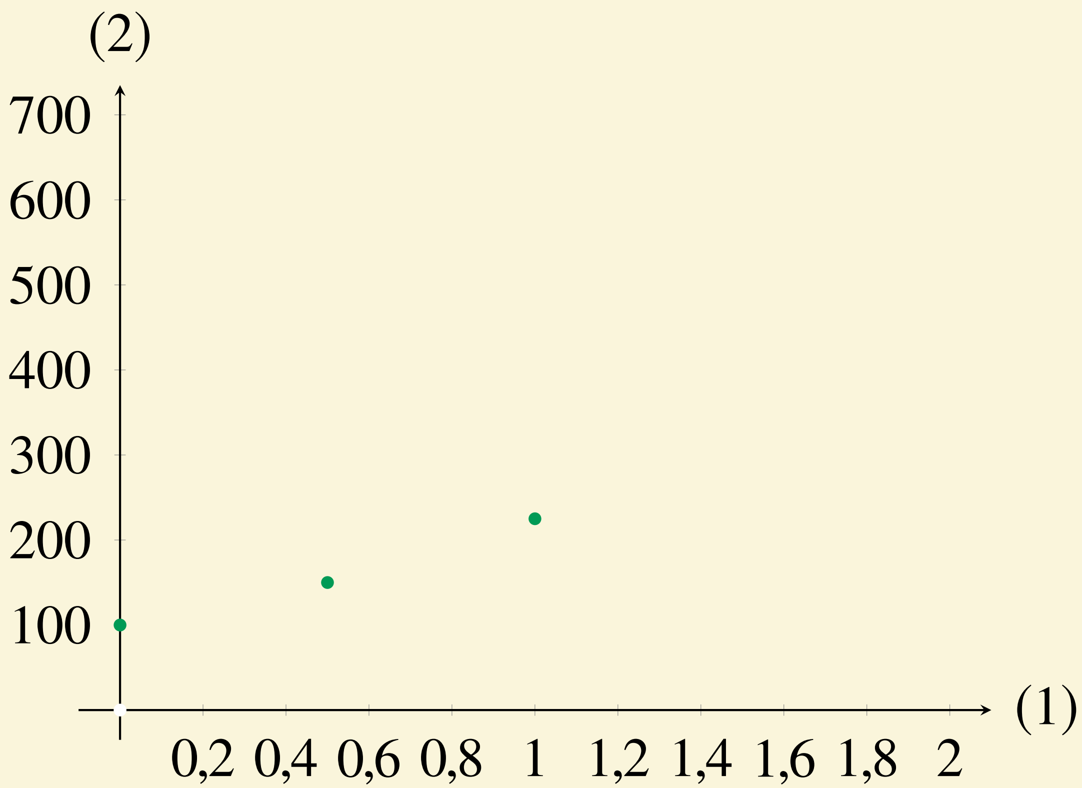

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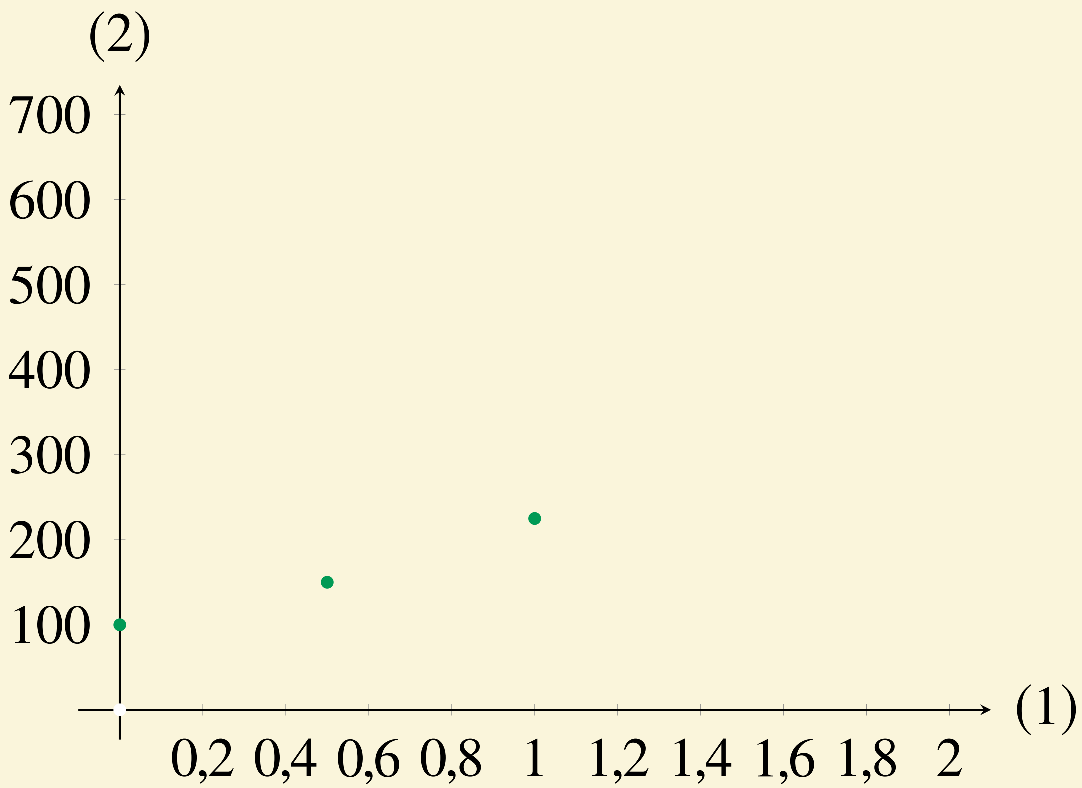

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Numerisk løsning af differentialligning

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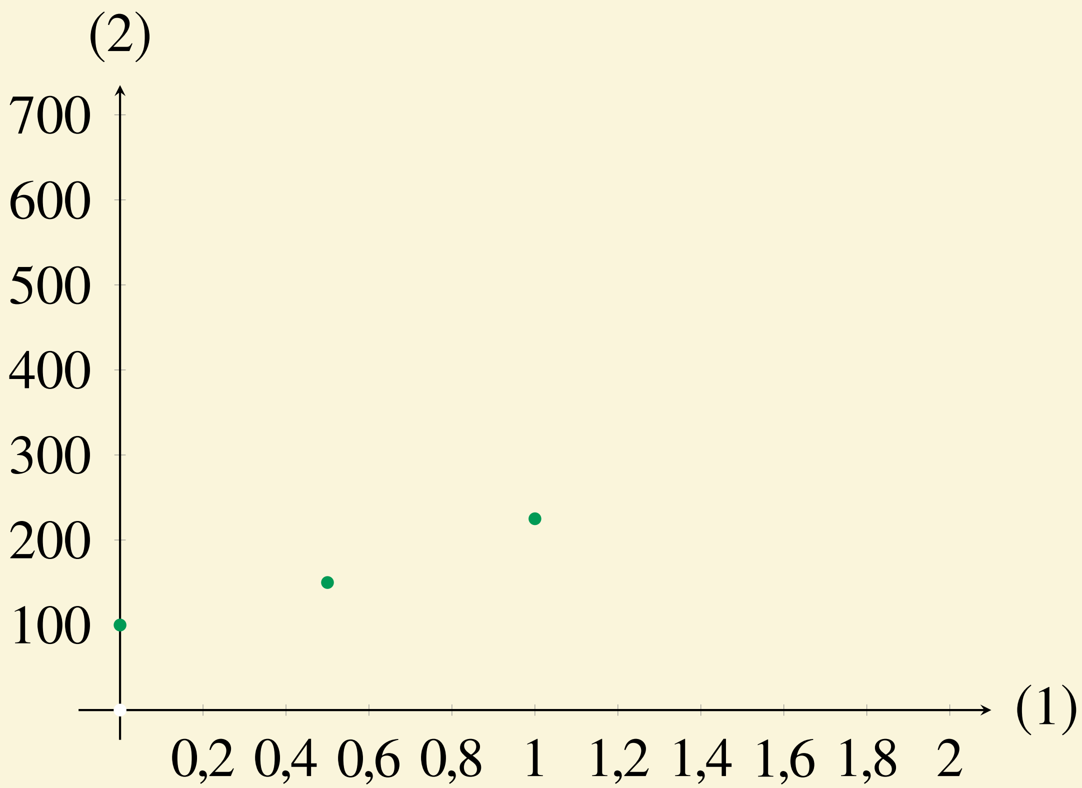

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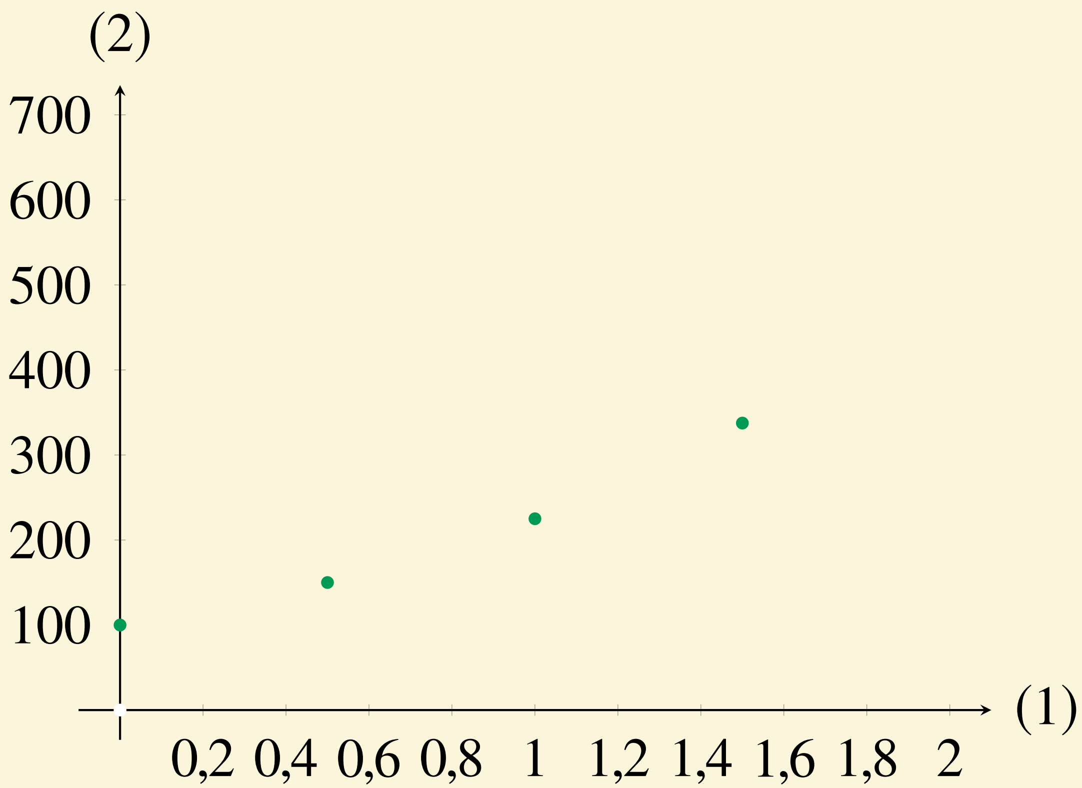

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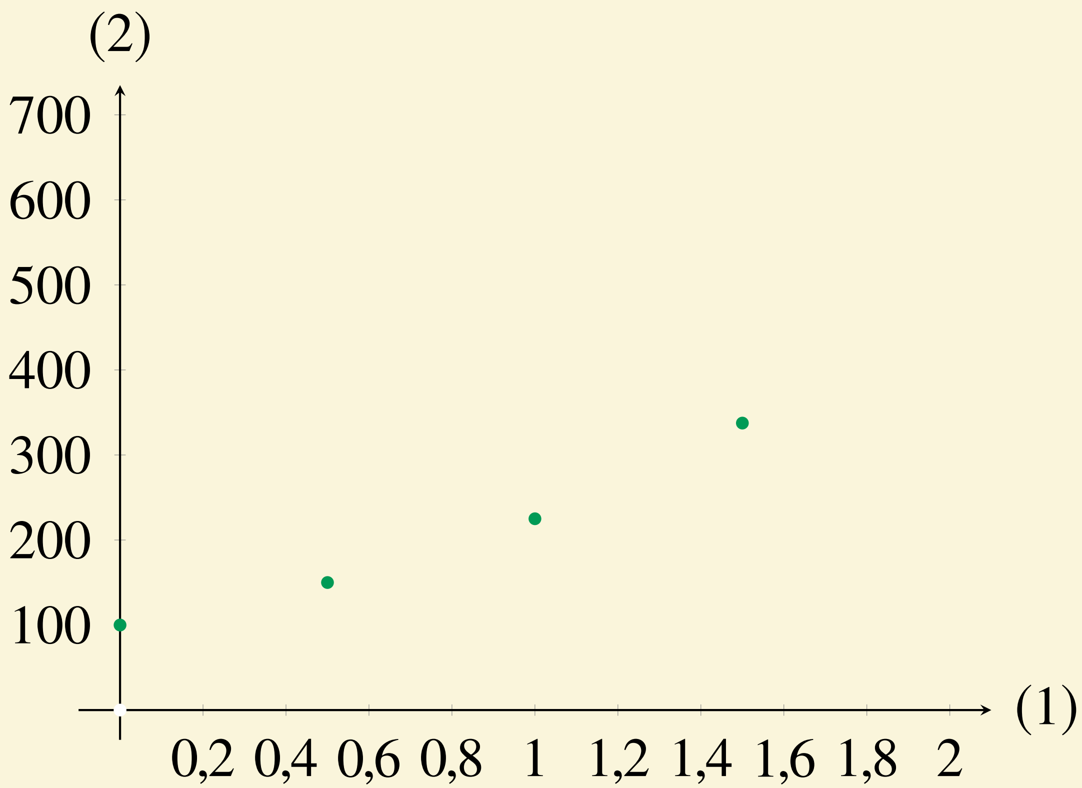

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Numerisk løsning af differentialligning

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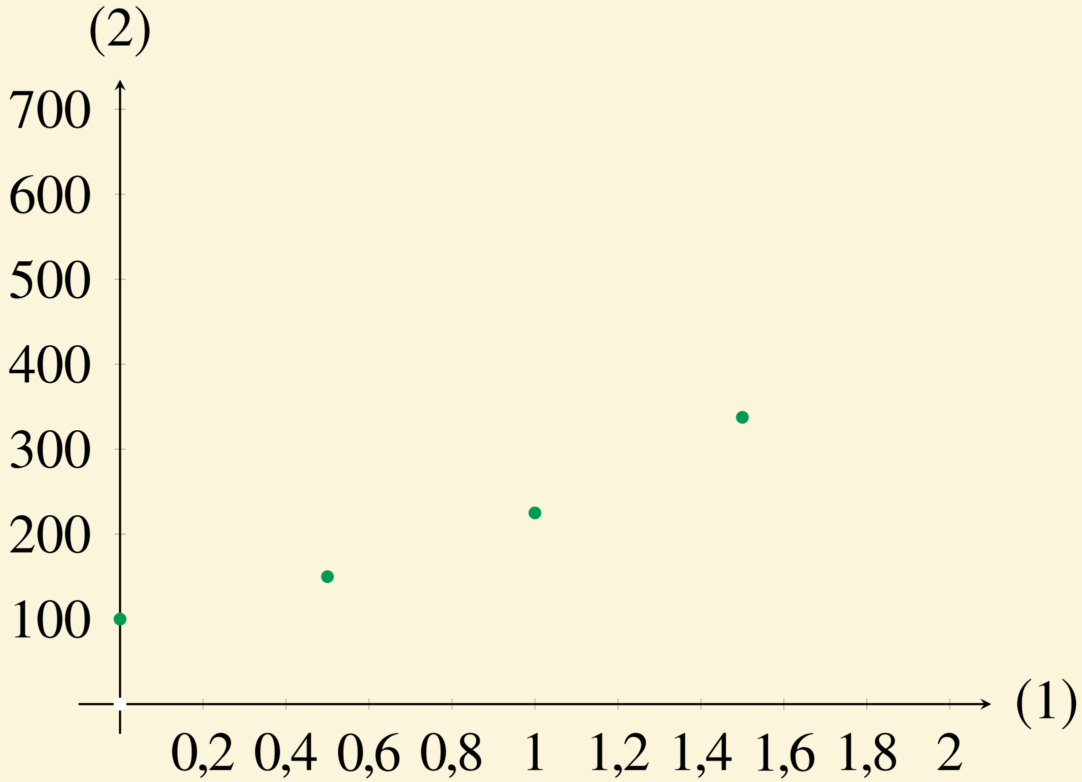

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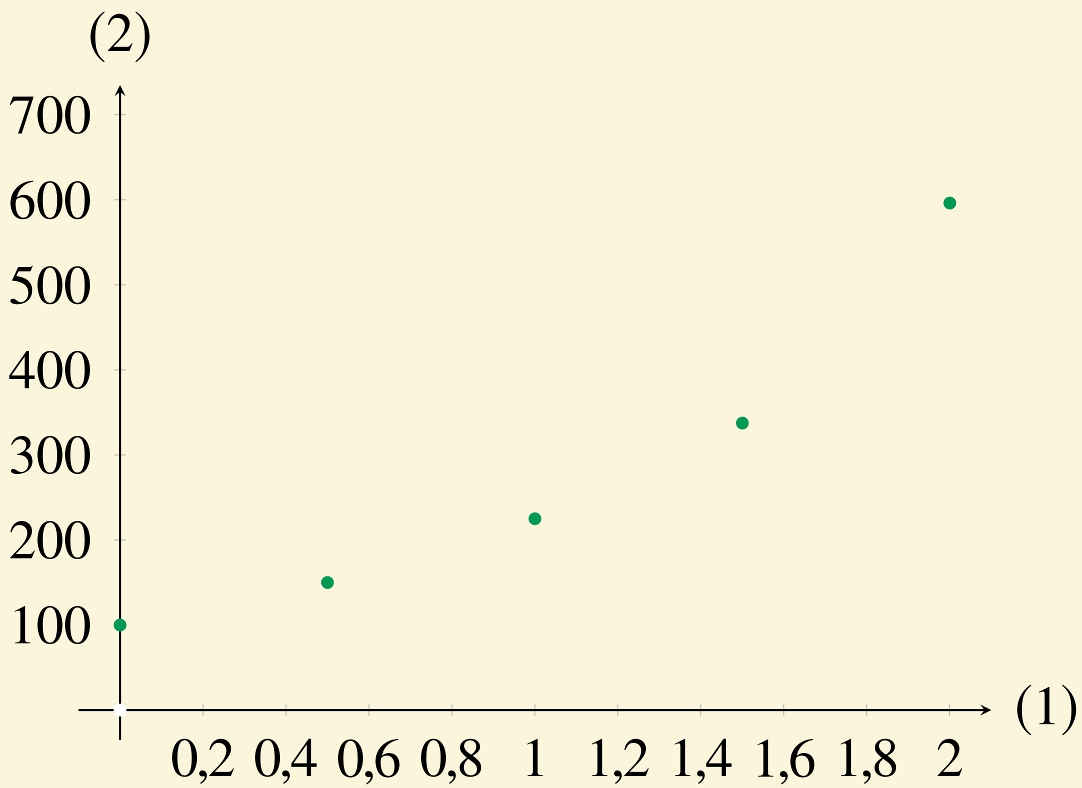

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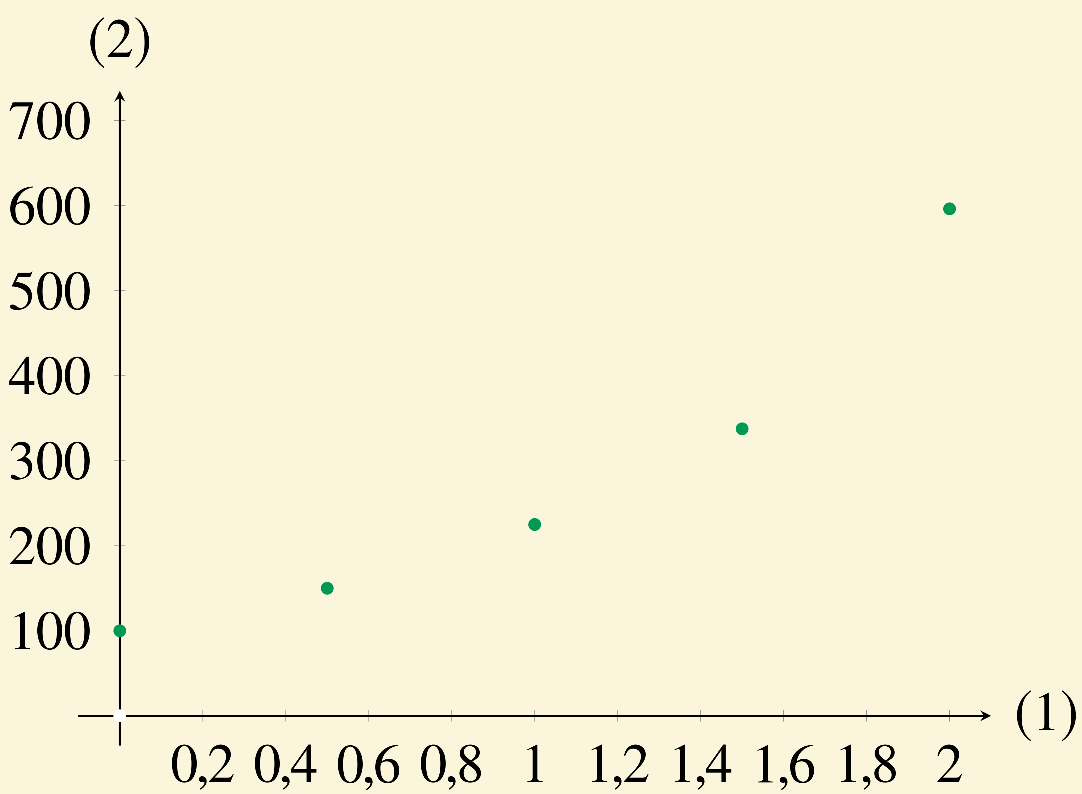

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Fra den analytiske løsning ved vi, at renten er $r = e^1 - 1$ som i dette tilfælde er $e^1 - 1 \approx 1.72$.

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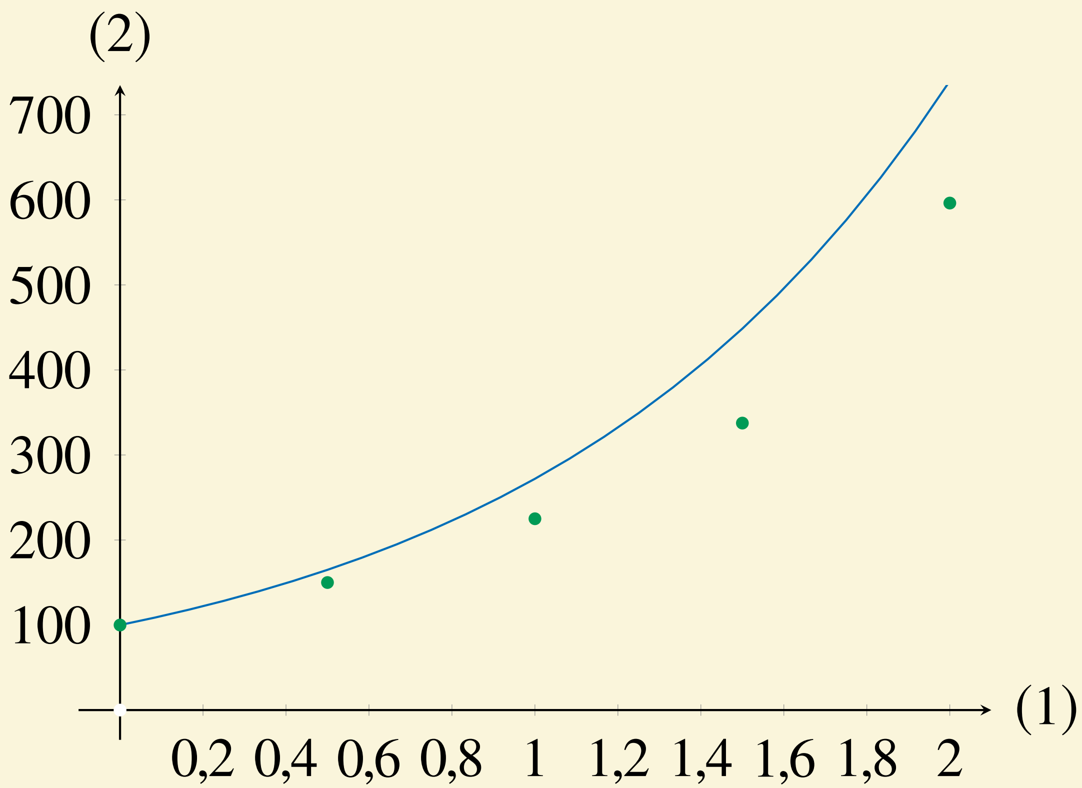

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